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A Mathematical Model of Velocity of Avian Flight

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ABSTRACT: A mathematical model is deduced for the computation of forward velocity in horizontal avian flight. The data is presented for forward velocity of 25 species of small, medium and large size flying birds in their horizontal flight as a function of angle of attack, and wind velocity blown in the direction and also in opposite direction to that of bird flight. The present study concludes that the proposed theoretical model is the most suitable to explain the horizontal forward flight of natural fliers like birds. Further, it emphasises that single mechanism of wing beat controls, maintains and regulates the style, adaptation and performance of avian flight.

KEYWORD: Flying birds, Velocity of flight, Angle of attack, wind velocity, Horizontal flight.

I. INTRODUCTION

The forward flapping flight of birds is commonly observed in nature. There are two types of flights based on speed of the flight of a flier: (1). Fast forward flight (2). Slow forward flight. Fast forward flight is a normal mode of flight of majority of flying animals. In fast forward flight the wing always has some component of forward velocity and wing beat amplitudes are small, whereas in slow forward flight the wing beat amplitudes are large and the tips of the wings are coming to rest at the extremes of each stroke. In birds, the mechanism of flight is wing beat alone with more or less constant frequency and stroke amplitude unlike man-made airplanes. Any flying bird has to move forward, upward, downward and sometime backward according to its requirement.

Extensive work has been done on speed of the flier. Pennycuik [1] reported that apart from the wing beat frequency, the induced velocity of air is also a function of the body mass and wingspan of the flier. According to this theory the induced velocity of air should increase under transverse wing mutilation and should be independent of longitudinal wing mutilation. Tucker [2-5] made a systematic study of bird flight in a wind tunnel by changing the velocity of air and also by changing the inclination of the wind tunnel (by tilting). He succeeded in making metabolic measurements for different types of flight. Pennycuik [6] studied the gliding flight in the pigeon *Columba livia* through wind tunnel. Pennycuik [7] reviewed metabolic studies of the flight and he proposed some theoretical models for bird flight. Sanjay Sane and Nathaniel [8] in their study used a pair of hot wire anemometers to measure this flow at two locations near the body of a tethered flapping hawk moth, *Manduca sexta*. The axial inflow anemometer measured the airflow prior to its entry into the stroke plane, whereas the radial outflow anemometer measured the airflow after it crossed the stroke plane. The high temporal resolution of the hot wire anemometers allowed one to measure not only the mean induced flow but also subtle higher frequency disturbances occurring at 1-4 times the wing beat frequency. These data provided evidence for the predictions of a proposed mathematical model. The absolute value of the measured induced flow matched the estimate of the model. The experiments showed that wing flexion contributed significantly to the observed higher frequency disturbances. Thus, the hot wire anemometry technique provided a useful means to quantify the aerodynamic signature of wing flexion. MazherSaleem and Adeel Ahmad [9] constructed flight balance and a low speed wind tunnel in the laboratory and studied speed of the insect *Tesserautomajavanica* at different wind velocities ranging from 1 m/sec to 10 m/sec. This wind tunnel provided streamline flow of air and wind speed could be continuously varied. Bo Cheng [10] investigated experimentally the three dimensional vortex wake structure in both near and far field of a dynamically scaled flapping wing, using volumetric three – component velocimetry. They confirmed that convection and secondary tilting and stretching effects dominated the evolution of vorticity.

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NasihaSaherBano, et al [11] presented a comparative study on wing beat frequency of various species of birds. Frequency of wing beat of avian fliers was calculated using different theories and the data was compared and discussed. They suggested that Mass flow theory was superior to any other theory proposed for the computation of wing beat frequency. NasihaSaherBano, et al

[12] reported data on induced power, inertial power and dynamical efficiency of 25 species of wide variety of birds, when they fly in the state of hovering. Also, presented body parameters such as body mass, length, span, effective breadth, area, moment of inertia of the wing and frequency of wing beat of birds. The study on flight of small, medium and large size birds revealed that induced power was directly proportional to mass or weight of the flier. Further, suggested that inertial power was not a function of any one parameter of flight surface (wing) of a flier, but it comprised wing dimensions, moment of inertia, stroke angle and frequency of wing beat. Finally, they concluded that dynamic efficiency (η) of flight surface (wing) of birds was a function of body parameters as such it could not be related to a single parameter and was independent of size (small, medium and large) of a flier.

In the present study, a simple mathematical model is built for the computation of velocity of avian flight at its different states of flight (Hovering and horizontal flapping) in still air and also at different wind velocities and directions.

II. A MATHEMATICAL MODEL

Consider Fig.1., showing body axis, wing plane axis and the direction of velocity V_i of air induced due to wing beat, when the flier is flying with the velocity V in the forward direction in still air ($V_a = 0$). Let γ be the angle between wing plane axis and body axis as shown in Fig.1. Let ϕ be the angle of attack i.e., the angle made by body axis with the horizontal (x-axis). θ is the angle made by the induced velocity vector with y-axis. The components of V_i along x and y axis are:

$$\begin{aligned} V_{ix} &= V_i \sin \theta; \\ V_{iy} &= V_i \cos \theta \end{aligned}$$

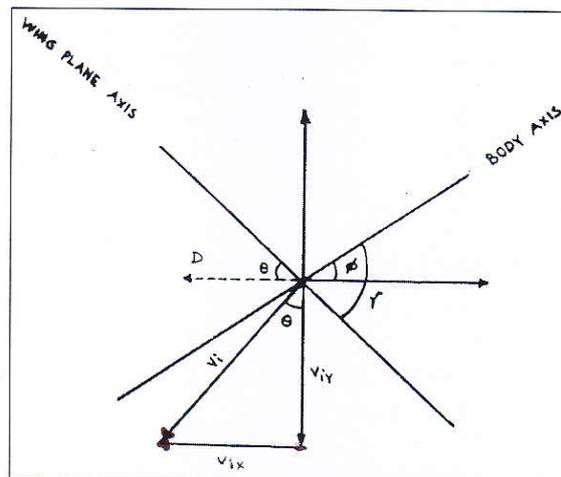


Fig. 1. Vector diagram of a bird and velocity

But the horizontal component V_{ix} of induced air V_i is equal and opposite to the forward velocity V of the flier, ie $-V_x = V$.

Hence $V = V_i \sin \theta$ (1)

From Fig. 1.

$$\theta = \gamma - \phi$$

Therefore, $V = V_i \sin (\gamma - \phi)$ (2)

For birds and bats

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$$\begin{aligned} \gamma &= 90^\circ \\ V &= V_i \sin(90^\circ - \phi) \end{aligned} \tag{3}$$

For insects

$$\begin{aligned} \gamma &= 60^\circ \\ V &= V_i \sin(60^\circ - \phi) \end{aligned} \tag{4}$$

V is calculated by changing the angle of attack ϕ in the interval of 10° and data is tabulated in Table 1.

2.1. Calculation of forward velocity of a bird in its horizontal flight at different wind Velocities

(a). Consider a flier flying with the velocity V' in the direction of wind blowing with a velocity V_a . Fig. 2. shows the vector diagram of velocity of a bird flying in the direction of wind.

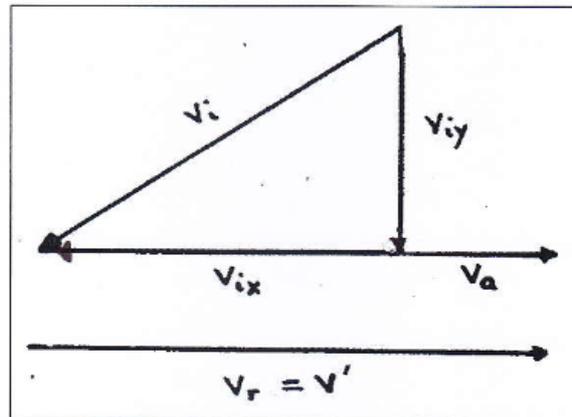


Fig. 2. Vector diagram of velocity of a bird flying in the direction of wind

From the figure,

$$V_r = -V_{ix} + V_a$$

Since

$$-V_{ix} = V; V_r = V + V_a$$

$$V' = V_i \sin \theta + V_a \tag{5}$$

where $V_r = V'$, the forward velocity of the flier; $\theta = \gamma - \phi$

(b). Consider a flier flying with the velocity V' in the direction opposite to the direction of wind blowing with the velocity V_a . Fig. 3. shows vector diagram of velocity of a bird flying in the opposite direction of wind.

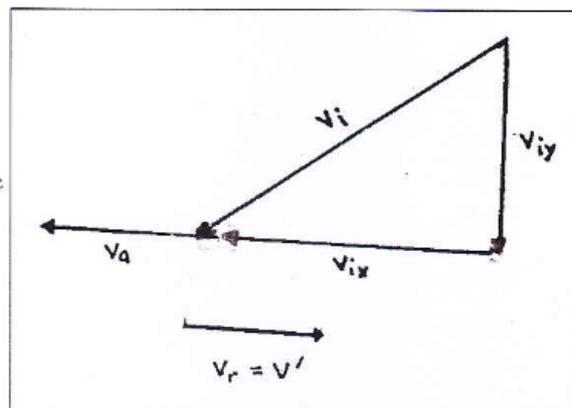


Fig. 3. Vector diagram of velocity of a bird flying in the opposite direction of wind

From the figure

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$$\sin \theta = \frac{V_{ix}}{V_i};$$

$$V = -V_{ix} = V_i \sin \theta$$

But

$$V_r = V - V_a$$

$$V' = V_i \sin \theta - V_a, \text{ where } V_r = V', \text{ the forward velocity of the flier} \quad (6)$$

The forward velocity of the birds has been calculated by assigning different values to V_a with an interval of 100 cm/sec. For the purpose, of calculations γ has been taken as 90° for birds.

III. RESULTS AND DISCUSSION

Table 1 gives data on forward velocity of horizontal flight for 25 species of birds in still air at different angles of attack ranging from 10° to 90° with an interval of 10° . It is interesting to note that the velocity of birds decreases non-linearly with the increase in the angle of attack. When the angle of attack is 90° , the wing stroke plane becomes horizontal and velocity of fliers attains to zero. Actually this is the condition for hovering flight in which the flier can not move forward and will be in dynamical equilibrium, beating its wings.

Table 1 - Velocity of birds at different angle of attack

Flier	V_i (cm/sec)	Velocity (cm.sec ⁻¹) of the flier at the angle of attack (degree) of								
		$\phi = 10$	20	30	40	50	60	70	80	90
Passeriformes										
<i>Nactoriniaasiatica</i>	110	108	103	95	84	71	55	38	19	0
<i>Estrildaamandava</i>	135	133	127	117	104	87	69	46	24	0
<i>Loncuramalabarica</i>	142	140	133	123	109	91	71	49	25	0
<i>Loncurepunculata</i>	141	139	133	122	108	91	71	48	25	0
<i>Loncuramalacca</i>	170	167	160	147	130	109	85	58	30	0
<i>Passer domesticus</i>	151	148	142	130	115	97	76	52	26	0
<i>Dicrurusadsimilis</i>	111	109	104	96	85	71	55	38	19	0
<i>Turdoidesstriatus</i>	161	159	151	140	123	104	81	55	28	0
<i>Acridotherustristis</i>	149	146	140	129	114	96	74	51	26	0
<i>Corvussplendens</i>	149	146	140	129	114	96	74	51	26	0
Psittaciformes										
<i>Melosittacusundulatus</i>	144	142	135	125	110	93	72	49	25	0
<i>Psittaculacathorpeae</i>	129	128	122	112	99	83	65	44	23	0
<i>Psittaculakrameri</i>	145	143	136	126	111	93	72	50	25	0
Pelecaniformes										
<i>Egerettagarzetta</i>	148	146	140	129	114	95	74	51	26	0
<i>Ardeolagravii</i>	160	158	150	138	123	103	80	55	28	0
<i>Bubulauis ibis</i>	136	134	128	118	104	87	68	46	24	0
Coraciiformes										
<i>Meropsorientalis</i>	100	99	94	87	77	64	50	34	17	0
<i>Upupaapops</i>	110	108	104	95	84	71	55	38	19	0
<i>Coraciasindica</i>	124	122	117	107	95	80	62	42	22	0
Apodiformes										
<i>Apusaffinus</i>	100	99	94	87	77	64	50	34	17	0
Galliformes										
<i>Peridiculaastatica</i>	156	154	147	135	119	100	78	54	27	0
Strigiformes										
<i>Bubo bengalensis</i>	154	152	145	134	118	99	77	53	27	0

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Falconiformes										
<i>Falco peregrinus</i>	148	146	139	128	114	95	74	51	26	0
Gruiformes										
<i>Grusgrus</i>	185	182	174	160	142	119	92	63	32	0
Columbiformes										
<i>Columba livia</i>	177	174.	167	153	136	114	88	60	30	0

V_i : velocity of induced air ($\text{cm}\cdot\text{sec}^{-1}$).

Table 2 presents data on forward velocity of horizontal flight of birds as a function of velocity of wind blown in the direction of flight of the bird. From Table 2, it is evident that forward velocity of a flier increases significantly with the wind velocity. The dependence of forward velocity on wind velocity is linear.

Table 2 - Velocity of birds flying in the direction of wind

Flier	Velocity ($\text{cm}\cdot\text{sec}^{-1}$) of the flier, when air blown in the same direction with velocity ($\text{cm}\cdot\text{sec}^{-1}$) of										
	0	100	200	300	400	500	600	700	800	900	1000
Passeriformes											
<i>Nactoriniaasiatica</i>	110	210	310	410	510	610	710	810	910	1010	1110
<i>Estrildaamandava</i>	135	235	335	435	535	635	735	835	935	1035	1135
<i>Loncuramalabarica</i>	142	242	342	442	542	642	742	842	942	1042	1142
<i>Loncurepunculata</i>	141	241	341	441	541	641	741	841	941	1041	1141
<i>Loncuramalacca</i>	170	270	370	470	570	670	770	870	970	1070	1170
<i>Passer domesticus</i>	151	251	351	451	551	651	751	851	951	1051	1151
<i>Dicrurusadsimilis</i>	111	211	311	411	511	611	711	811	911	1011	1111
<i>Turdoidesstriatus</i>	161	261	361	461	561	661	761	861	961	1061	1161
<i>Acridotherustristis</i>	149	249	349	449	549	649	749	849	949	1049	1149
<i>Corvussplendens</i>	149	249	349	449	549	649	749	849	949	1049	1149
Psittaciformes											
<i>Melosittacusundulatus</i>	144	244	344	444	544	644	744	844	944	1044	1144
<i>Psittaculacathorpeae</i>	129	229	329	429	529	629	729	829	929	1029	1129
<i>Psittaculakrameri</i>	145	245	345	445	545	645	745	845	945	1045	1145
Pelecaniformes											
<i>Egerrettagarzetta</i>	148	248	348	448	548	648	748	848	948	1048	1148
<i>Ardeolagravii</i>	160	260	360	460	560	660	760	860	960	1060	1160
<i>Bubulaus ibis</i>	136	236	336	436	536	636	736	836	936	1036	1136
Coraciiformes											
<i>Meropsorientalis</i>	100	200	300	400	500	600	700	800	900	1000	1100
<i>Upupaapops</i>	110	210	310	410	510	610	710	810	910	1010	1110
<i>Coraciasindica</i>	124	224	324	424	524	624	724	824	924	1024	1124
Apodiformes											
<i>Apusaffinus</i>	100	200	300	400	500	600	700	800	900	1000	1100
Galliformes											
<i>Peridiculaastatica</i>	156	256	356	456	556	656	756	856	956	1056	1156
Strigiformes											
<i>Bubo bengalensis</i>	154	254	354	454	554	654	754	854	954	1054	1154
Falconiformes											
<i>Falco peregrinus</i>	148	248	348	448	548	648	748	848	948	1048	1148
Gruiformes											
<i>Grusgrus</i>	185	285	385	485	585	685	785	885	985	1085	1185

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Columbiformes											
<i>Columba livia</i>	177	277	377	477	577	677	777	877	977	1077	1177

Table 3 shows data on forward velocity of horizontal flight of birds as a function of velocity of wind blown in the direction opposite to that of flight of the bird. From the table, it can be noted that forward velocity of a flier decreases linearly with the increase in wind velocity.

Table 3 - Velocity of birds flying in the direction opposite to wind direction

Flier	Velocity (cm.sec ⁻¹) of the flier, when air blown in the opposite direction with velocity (cm.sec ⁻¹) of										
	0	10	20	30	40	50	60	70	80	90	100
Passeriformes											
<i>Nactoriniaasiatica</i>	110	100	90	80	70	60	50	39	30	20	10
<i>Estrildaamandava</i>	135	125	115	105	95	85	75	65	55	45	35
<i>Loncuramalabarica</i>	142	132	122	112	102	92	82	72	62	52	42
<i>Loncurepunculata</i>	141	131	121	111	101	91	81	71	61	51	41
<i>Loncuramalacca</i>	170	160	150	140	130	120	110	100	90	80	70
<i>Passer domesticus</i>	151	141	131	121	111	101	91	81	71	61	51
<i>Dicrurusadsimilis</i>	111	101	91	81	71	61	51	41	31	21	11
<i>Turdoidesstriatus</i>	161	151	141	131	121	111	101	91	81	71	61
<i>Acridotherustristis</i>	149	139	129	119	109	99	89	79	69	59	49
<i>Corvussplendens</i>	149	139	129	119	109	99	89	79	69	59	49
Psittaciformes											
<i>Melosittacusundulatus</i>	144	134	124	114	104	94	84	74	64	54	44
<i>Psittaculacathorpae</i>	129	119	109	99	89	79	69	59	49	39	29
<i>Psittaculakrameri</i>	145	135	125	115	105	95	85	75	65	55	45
Pelecaniformes											
<i>Egerettagarzetta</i>	148	138	128	118	108	98	88	78	68	58	48
<i>Ardeolagravii</i>	160	150	140	130	120	110	100	90	80	70	60
<i>Bubulaus ibis</i>	136	126	116	106	96	86	76	66	56	46	36
Coraciformes											
<i>Meropsorientalis</i>	100	90	80	70	60	50	40	30	20	10	10
<i>Upupaapops</i>	110	100	90	80	70	60	50	40	30	20	10
<i>Coraciasindica</i>	124	114	104	94	84	74	64	54	44	34	24
Apodiformes											
<i>Apusaffinus</i>	100	90	80	70	60	50	40	30	20	10	10
Galliformes											
<i>Peridiculaastatica</i>	156	146	136	126	116	106	96	86	76	66	56
Strigiformes											
<i>Bubo bengalensis</i>	154	144	134	124	114	104	94	84	74	64	54
Falconiformes											
<i>Falco peregrinus</i>	148	138	128	118	108	98	88	78	68	58	48
Gruiformes											
<i>Grusgrus</i>	185	175	165	155	145	135	125	115	105	95	85
Columbiformes											
<i>Columba livia</i>	177	167	157	147	137	127	117	107	97	87	77

As far as forward velocity of horizontal flight of flier is concerned, the mathematical model suggests that the induced velocity of air is species specific along with body parameters, the horizontal component of which is the

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forward velocity of a flier. Whatever may be the style of flight, whether hovering or forward flight, induced velocity remains to be constant as it is the function of frequency of wing beat: affective wing breadth; and acceleration due to gravity (Adeel Ahmad, 1978). Hovering or forward flight ceases when induced velocity becomes zero. As the forward velocity is the horizontal component of induced velocity, it changes with the angle of attack of the flier. Hence, a flier can have forward horizontal flight with different velocities based on the change in angle of attack. It means if a flier were to fly with a desired forward velocity, it has to change its angle of attack. But according to Pennycuick (1968) in forward flight weight must be balanced by the rate of change of downward momentum imparted to the air. The mass flow of air through the wing disc is the product of the density of air, the disc area and resultant of the induced velocity and the forward velocity. The question is, how does the flier get the forward velocity? This aspect is not justified from Pennycuick model. Hence,

this well established model (Pennycuick's model) is inadequate to explain the phenomenon of forward flight of a natural flier. On the other hand the proposed model of the present investigation is the most suitable one in this situation and suggests that:

The induced velocity of air is the same, irrespective of style of flight, and horizontal component of which is the forward velocity, regulated by the angle of attack of the flier.

IV. CONCLUSIONS

1. The proposed theoretical model is the most suitable to explain the horizontal forward flight of natural fliers like birds.
2. Single mechanism of wing beat controls, maintains and regulates the style, adaptation, performance of avian flight.
3. The present model of velocity can be used for the calculation of velocity of fliers flying at high altitudes because of the fact that V_i is a function of density and acceleration due to gravity which change at large heights.

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